INFLATION AND WELFARE IN INTERNATIONAL TRADE

by

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CORE DISCUSSION PAPER N° 8601

CENTER FOR OPERATIONS RESEARCH & ECONOMETRICS

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December 1985

A B S T R A C T

The paper demonstrates the role of inflation as one of the major causes of inefficiencies in international trade. In a model of international trade among two overlapping generations economies it is shown that opening up trade between the two countries may change an efficient autarchic situation to an inefficient free trade equilibrium and that all agents of one country are made worse off under free trade. Furthermore, inefficient international competitive equilibria can always be improved upon through price distortions and/or quantity constraints, but never by choosing a constant exchange rate.

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1. INTRODUCTION

Among the many fundamental insights of the pure theory of international trade one finds the result that free trade among any two countries can be no worse than autarchy and that free trade equilibria are Pareto efficient (e.g. Dixit/Norman 1980). The first statement which implies universal gains from trade has to be modified appropriately if there is a diversity of consumers. However, all results on gains from trade hold true for an arbitrary finite number of commodities and agents. This yields in particular efficiency results for models with intertemporal trade and/or uncertainty with the appropriate redefinition of commodities. Formally speaking, all of these results are special cases of the fundamental welfare theorem. Inefficiencies in trade are therefore mostly attributed to market imperfections such as quotas, tariffs, currency restrictions, exchange rate or price rigidities.

In contrast to the pure theory of international trade, monetary theory treating questions involving money and other assets, interest rates, and inflation, essentially requires an infinite setting, since finite economies will not assign positive competitive equilibrium values or prices to assets in general. It has been known for some time (Samuelson (1958)) that the fundamental theorem of welfare economics fails in general in infinite economies. For the case with international trade Gale (1971) in a seminal paper indicated the fundamental difference of efficiency of free trade equilibria in infinite economies to the standard finite case of the pure theory. It is the purpose of this paper to investigate further the sources of inefficiency in international trade and provide some structural insight. According to our present understanding this can be done within the framework of a stationary two country world economy where each country consists of a simple overlapping generations exchange economy. In order to keep the presentation as simple as possible production and population growth are excluded. But they could be included in a straightforward manner without altering the results of the paper in a fundamental way. The paper provides a structural characterization of the sources of inefficiencies in international trade. Moreover, we describe institutions or policies which can be used to Pareto improve competitive international equilibria. More specifically, it is shown that opening up trade between two countries may change an efficient autarchic situation to an inefficient free trade equilibrium and that all agents of one country are made worse off under free trade. Furthermore, inefficient international competitive equilibria can always be improved upon through price distortions and/or quantity constraints, but never by choosing a constant exchange rate.

The literature of the past decade includes some attempts at the issues treated in this paper, notably Buiter (1981), Dornbusch (1985), and Persson (1985). Most of these contributions, however, use a much more elaborate asset structure as well as active government policies, capital formation, and population growth. Since, in our view, these additional features seem to cloud the essential sources of the failure of Pareto efficiency, it is hoped that the rigorous and complete treatment of the simple stationary exchange case will help to bridge the gap between pure and monetary trade theory.

2. THE MODEL

We shall consider a world consisting of two countries, each of which is described by an overlapping generations model. Generations live for two consecutive time periods, and for simplicity we assume that there is only one good available in each time period and only one consumer in each generation. The two economies are described in detail below. To facilitate notation we employ lower case letters for variables and parameters of Economy 1 and upper case letters for Economy 2.

ECONOMY 1. There is a countable infinity $t = 1,2,\ldots$ of time periods and a single good available for consumption in each period. For each t there is a consumer ("the agent born at time t") who consumes in periods t and $t+1$. Consumer t is characterized by his *consumption set* X_t, his *utility function* $u_t : X_t \to \mathbb{R}$, and his *initial endowment* $e_t \in X_t$.

To get as simple and transparent a model as possible, we assume that the consumers $t = 1,2,\ldots$ are all identical with $X_t = \mathbb{R}^2_+$, $u_t = u$ strictly monotone, strictly quasi-concave and differentiable. Moreover, we shall be rather explicit on the initial endowments:

ASSUMPTION e. *The initial endowment of every consumer* $t \geq 1$ *in* ECONOMY 1 *is such that* $e \neq 0$ *and*

$$-u_1'(e) + u_2'(e) > 0.$$

Recalling that $u'(e) = (u_1'(e), u_2'(e))$ is the gradient of the utility function and $e = (e_1, e_2)$, we see that the vector $(-1,1)$ points into the preferred set at the point e. The assumption prescribes a certain relationship between preferences and endowments. With his initial endowments, the consumer will want to trade at the prices $(p_1,p_2) = (1,1)$, giving up some consumption of his endowment in the first period of his life in return for the same amount of the good in the second ("the consumer wants to save for his old age").

To complete the model, we need an initial zero'th consumer living only in period 1. This consumer is characterized by a consumption set $X_o = \mathbb{R}_+$, a utility function u_o on $\mathbb{R}_+$, which may be taken to be the identy map, and an initial endowment $e_o \in \mathbb{R}_+$. Finally, we endow the initial consumer with a positive amount m of money. We shall see that the presence of this initial money endowment is needed if there are to be interesting equilibria in the model.

A *competitive equilibrium* in ECONOMY 1 is a pair $\left((x^t)_{t=0}^{\infty},(p_t)_{t=1}^{\infty}\right)$, where for each $t \geqq 0$, x^t is a feasible consumption for consumer t, and for each $t \geqq 1$, $p_t > 0$ is the price of commodity t, such that

(i) individual utility maximization:

(a) for $t = 0$, x^0 maximizes u_o subject to $p_1 x^0 \leqq p_1 e_o + m$,

(b) for $t \geqq 1$, x^t maximizes u subject to

$$p_t x_t^t + p_{t+1} x_{t+1}^t \leqq p_t e_t^t + p_{t+1} e_{t+1}^t.$$

(ii) feasibility: For each $t \geqq 1$,

$$x_t^{t-1} + x_t^t = e_t^{t-1} + e_t^t.$$

A competitive equilibrium is *quasi-stationary* if $x^t = x^{t'} = x$, all $t,t' \geqq 1$, and *stationary* if it is quasi-stationary and $p_t = p_{t'} = p$, all $t,t' \geqq 1$.

PROPOSITION 1. If ECONOMY 1 *satisfies* ASSUMPTION e, *then there is a stationary equilibrium with* $x \neq e$.

Proof:
Consider the problem Max $\{u(x) | x_1 + x_2 \leq e_1 + e_2\}$. By our assumptions on the utility function, this problem has a unique solution x with $x_1 + x_2 = e_1 + e_2$. ASSUMPTION e implies that $x \neq e$, since the set $\{x' | x'_1 + x'_2 = e_1 + e_2\}$ contains points for which $u(x') > u(e)$.

Let $x^0 = e_1 - x_1 > 0$, and let $x^t = (x_1, x_2)$, $t \geqq 1$. Then $(x^t)_{t=0}^{\infty}$ satisfies the feasibility condition (ii). Defining $(p_t)_{t=1}^{\infty}$ by $p_1 = p_2 = \ldots = p = m/(e_1 - x_1)$, it is easily seen that we have a stationary competitive equilibrium. □

In the remainder of this section on ECONOMY 1, we introduce some auxiliary concepts and some additional assumptions which will allow us to give an intuitive graphical illustration of stationary and quasi-stationary equilibria and of the points to be raised in the following.

Define the *excess demand* $z(p_t, p_{t+1}) \in \mathbb{R}^2$ of consumer t for $t \geqq 1$ by the condition that $z(p_t, p_{t+1}) + (e_t, e_{t+1}) = (x_t, x_{t+1})$ solves the problem

$$\text{(1)} \qquad \begin{array}{l} \text{Max } u(x_t, x_{t+1}) \\ \text{subject to} \\ p_t x_t + p_{t+1} x_{t+1} \leqq p_t e_t + p_t e_{t+1}. \end{array}$$

Then $z(p_t, p_{t+1})$ satisfies Walras' Law $p_t z_1(p_t, p_{t+1}) + p_{t+1} z_2(p_t, p_{t+1}) = 0$ and is continuous at all $(p_t, p_{t+1}) \in \mathbb{R}^2_{++}$. Moreover, z depends only on relative prices which we define as $\theta_t = p_t / p_{t+1}$. The first coordinate of $z(p_t, p_{t+1})$, considered as a function of relative prices only, is written as $\tilde{s}(\theta_t)$ and is called *excess supply*. For the graphical arguments, we assume throughout that $\tilde{s}(\theta_t)$ is non-positive, that $\tilde{s}(\theta_t) = 0$ for some $\theta_t > 0$ and that $\tilde{s}$ is C^1 and strictly decreasing. Let $s = \tilde{s}^{-1}$ be inverse excess supply, i.e.

$$s(z_t) = \{\theta_t \mid z_t = \tilde{s}(\theta_t)\},$$

then s is C^1 and strictly decreasing. We can define the *offer curve* as a function from supply in the first period z_t to demand in the second by

$$d(z_t) = -s(z_t)z_t.$$

Clearly $d : \mathbb{R}_- \to \mathbb{R}_+$ is C^1 and decreasing. It may look as the offer curve in Figure 1.

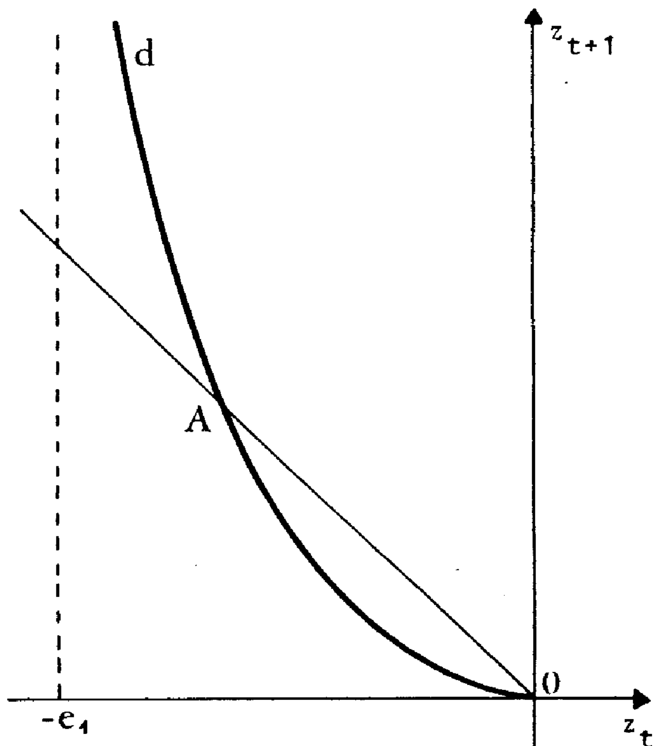

Figure 1

It should be noted that the assumptions made on the offer curve (in particular the monotonicity of excess supply) does not follow from standard assumptions on consumers' characteristics and indeed are violated in a great number of cases. Therefore the graphical arguments in this paper should be taken only as an illustration of the arguments behind the results.

If $\left((x^t)_{t=0}^{\infty}, (p_t)_{t=1}^{\infty}\right)$ is a competitive equilibrium then the sequence $(z_t)_{t=1}^{\infty}$ with $z_t = x_t^t - e_t^t$, $t \geqq 1$, satisfies

(a) $z_1 + m/p_1 = 0$

(b) $z_{t+1} + d(z_t) = 0 \qquad t \geqq 1.$

Conversely, let $(z_t)_{t=1}^{\infty}$ be a sequence satisfying (a) and (b) above, and define $(x^t)_{t=0}^{\infty}$ by $x^0 = e_0 - z_1$, $x^t = (z_t, d(z_t)) + e$, and let $(p_t)_{t=1}$ be defined by $p_1 = -m/z_1$, $p_{t+1} = p_t z_t / d(z_t)$. Then $\left((x^t)_{t=0}^{\infty}, (p_t)_{t=1}^{\infty}\right)$ is a competitive equilibrium. It follows that a stationary equilibrium is obtained where the offer curve intersects the half-line $\{\lambda(-1,1) \mid \lambda \geqq 0\}$ (point A in Figure 1). The existence of a stationary equilibrium in our graphical case is easily established. ASSUMPTION e implies $|d'(0)| < 1$, since $d'(0) = -s(0) > -1$, and $d(z_t) \to \infty$ as z_t tends to its minimum value.

ECONOMY 2. The second economy is similar to ECONOMY 1 in its construction. For each $t \geqq 1$, there is a consumer with consumption set $X_t = \mathbb{R}_+^2$, utility function $U_t : \mathbb{R}_+^2 \to \mathbb{R}$ and initial endowments $E_t = (E_t^t, E_{t+1}^t) \in \mathbb{R}_+^2$. All consumers t for $t \geqq 1$ are identical, but they differ from the corresponding consumers in ECONOMY 1.

ASSUMPTION E. *The initial endowment* $E = (E_1, E_2)$ *of every consumer* t *in* ECONOMY 2 *is such that* $E \neq 0$ *and*

$$U_1'(E) - U_2'(E) > 0.$$

ASSUMPTION E implies that moving from E in the direction (1,-1), that is giving up goods tomorrow in return for goods today, will result in a preferred consumption.

Finally, there is an initial consumer in ECONOMY 2, characterized by consumption set $X_o = \mathbb{R}_+$, utility function U_o on $\mathbb{R}_+$ (which again without loss of generality may be taken as the identity map) and an initial endowment $E_o > 0$. Finally, we assume that the initial consumer has a money debt (or a negative money endowment) $M < 0$ to be paid from the income $P_1 E_o$ in period 1. This somewhat unusual assumption, which parallels that of a positive money endowment in ECONOMY 2, will be commented upon at the end of this section.

A competitive equilibrium in ECONOMY 2 is a pair $\left((X^t)_{t=0}^{\infty}, (P_t)_{t=1}^{\infty}\right)$ such that

(i.a) for $t = 0$, X^0 maximizes U_o subject to $P_1 X^0 \leqq P_1 E_o + M$

(i.b) for $t \geqq 1$, X^t maximizes U subject to

$$P_t X_t^t + P_{t+1} X_{t+1}^t \leqq P_t E_t^t + P_{t+1} E_{t+1}^t$$

(ii) for each $t \geqq 1$,

$$X_t^{t-1} + X_t^t = E_t^{t-1} + E_t^t.$$

Quasi-stationarity and stationarity of equilibria are defined as above. We have the following counterpart of PROPOSITION 1, the proof of which is left to the reader:

PROPOSITION 2. *If* ECONOMY 2 *satisfies* ASSUMPTION E, *then there is a stationary equilibrium with* $X \neq E$.

For the graphical illustration of ECONOMY 2, we define excess demand $Z(P_t, P_{t+1})$ as for ECONOMY 1. Let $\tilde{S}(\Theta_t)$ be the first coordinate of $Z(P_t, P_{t+1})$ considered as a function of the price ratio $\Theta_t = P_t/P_{t+1}$. Assume that $\tilde{S}$ is C^1 and strictly decreasing with non-negative values, and $S(\Theta_t) = 0$ for some $\Theta_t > 0$. As above, we may define an *offer curve* $D : \mathbb{R}_+ \to \mathbb{R}_-$, considered as a function from demand Z_t in period t to supply in period t+1, by

$$D(Z_t) = - Z_t S(Z_t)$$

where $S = \tilde{S}^{-1}$. The offer curve is C^1 and strictly decreasing. It may look as the curve in Figure 2.

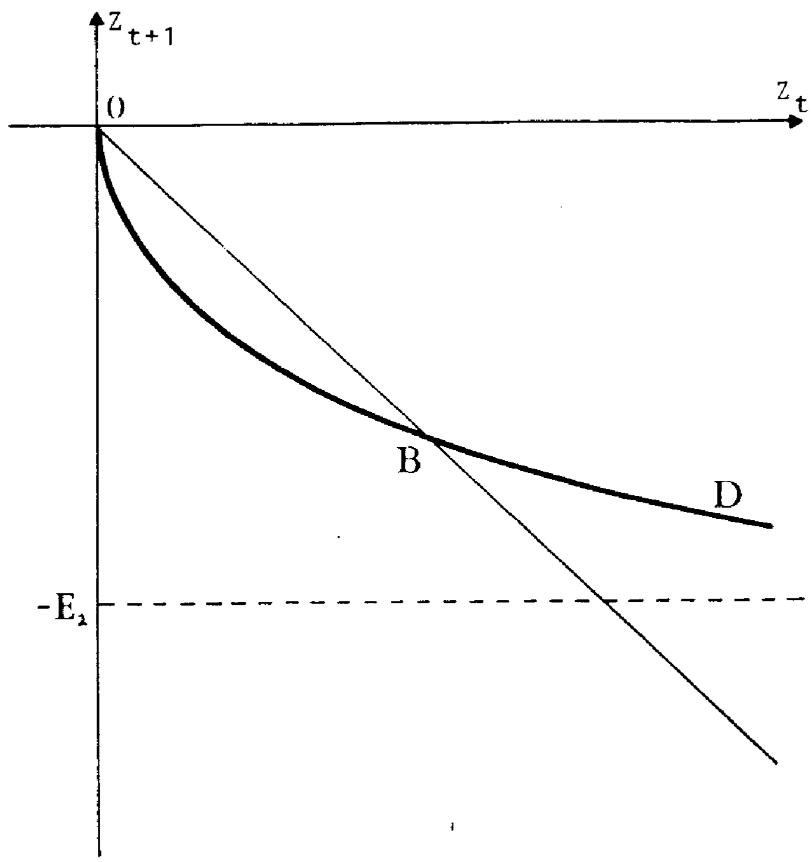

Figure 2

If $\left((X^t)_{t=0}^{\infty}, (P_t)_{t=1}^{\infty}\right)$ is a competitive equilibrium, then the sequence $(Z_t)_{t=1}$ with $Z_t = X_t^t - E_t^t$, $t \geqq 1$, satisfies

(A) $Z_1 + M/P_1 = 0$

(B) $Z_{t+1} + D(Z_t) = 0.$

Conversely, any such sequence $(Z_t)_{t=1}^{\infty}$ corresponds to a competitive equilibrium. It follows that a stationary equilibrium is obtained where the graph of D intersects the half-line $\{\lambda(1,-1) \mid \lambda > 0\}$ (point B in Figure 2).

The introduction of positive money endowments in ECONOMY 1 and negative money endowments - or the existence of debt - in ECONOMY 2 at period 1 needs some explanation. The principal reason for this assumption is that existence of a stationary equilibrium under autarchy would not obtain otherwise. This is rather easy to see. In ECONOMY 1 young consumers wanting to save in period 1 could not buy any durable commodity to represent this saving. In ECONOMY 2 young consumers wanting to borrow could not increase their consumption in period 1 anyway, since the old consumer would consume all of his endowment himself.

Opening up trade between the countries might remedy this situation. On the other hand, it would give us a result about the beneficial effects of trade too cheaply, that is to say, since free trade equilibrium would be superior to autarchy already by the fact that under autarchy only the situation where everybody stays at his initial endowment is a feasible allocation. We have chosen the present model with exogeneous money and debt, so that in each of the economies under consideration, equilibria under autarchy exist.

3. INTERNATIONAL COMPETITIVE EQUILIBRIA

In this and the following sections we consider the situation of trade between the two countries introduced in Section 2. As in the previous section we shall be mainly interested in quasi-stationary and stationary equilibria since these equilibria are the easiest to visualize.

Free trade between the two countries without restrictions of any kind implies that the two national price systems $(p_t)_{t=1}^{\infty}$ and $(P_t)_{t=1}^{\infty}$ must be identical up to the nominal rescaling given by the exchange rate between the two currencies. This essentially means that there is an international price system $(\pi_t)_{t=1}^{\infty}$ such that consumers of generation t, independent of their country of residence, may perform any net trade $\zeta^t = (\zeta_t^t, \zeta_{t+1}^t)$ satisfying $\pi_t\zeta_t^t + \pi_{t+1}\zeta_{t+1}^t \leqq 0$. In this case, the two domestic price systems $(p_t)_{t=1}^{\infty}$ and $(P_t)_{t=1}^{\infty}$ must conform to the international price system in the sense that

$$p_t/p_{t+1} = \theta_t = \pi_t/\pi_{t+1} = \Theta_t = P_t/P_{t+1}$$

for all $t \geqq 1$, i.e. relative prices (or, what in the present simple model amounts to the same thing, interest rates) are the same in the two countries. Moreover, the exchange rate ω_t at any time $t \geqq 1$ must be constant, since

$$\omega_t = \frac{p_t}{P_t} = \frac{\theta_t p_{t+1}}{\Theta_t P_{t+1}} = \frac{p_{t+1}}{P_{t+1}} = \omega_{t+1}.$$

Nominal prices, however, may differ due to the influence of the initial positive or negative endowment of domestic money in each country. For the definition of an international competitive equilibrium it is not necessary to write out both price systems separately. One can always choose $\pi_t = p_t$ for all $t \geqq 1$ and define the exchange rate appropriately.

A quadruple $\left((x^t)_{t=0}^{\infty}, (X^t)_{t=0}^{\infty}, (\pi_t)_{t=1}^{\infty}, \omega\right)$ is an *international competitive equilibrium* if

(i.a) $x^0(X^0)$ maximizes $u_o(U_o)$

subject to

$$\pi_1 x^0 \leqq \pi_1 e^0 + m$$

$$(\pi_1 X^0 \leqq \pi_1 E^0 + \omega M)$$

(i.b) for $t \geqq 1$

$x^t(X^t)$ maximizes $u(U)$

subject to

$$\pi_t x_t^t + \pi_{t+1} x_{t+1}^t \leqq \pi_t e_t^t + \pi_{t+1} e_{t+1}^t$$

$$\left(\pi_t X_t^t + \pi_{t+1} X_{t+1}^t \leqq \pi_t E_t^t + \pi_{t+1} E_{t+1}^t\right)$$

(ii) for $t \geqq 1$

$$x_t^{t-1} + X_t^{t-1} + x_t^t + X_t^t = e_t^{t-1} + E_t^{t-1} + e_t^t + E_t^t \quad \text{for all } t \geqq 1$$

An international equilibrium is called *quasi-stationary* if $x^t = x^{t'}$, $X^t = X^{t'}$ for all $t, t' \geqq 1$, and it is called *stationary* if also $\pi^t = \pi^{t'}$ for all $t, t' \geqq 1$.

Comparing the definition of an international competitive equilibrium with those of autarchic competitive equilibria in the preceding section, we note that if each economy has a stationary equilibrium (say x and X), then there is a stationary international equilibrium with bundles (x,X). Such equilibria are trivial in the sense that the two countries do not trade (or at least need not trade) with each other. Moreover, a stationary international equilibrium will typically be of this kind. In order to have international equilibria with non-zero international trade we must consider the larger class of quasi-stationary equilibria. The following theorem provides a structural characterization of quasi-stationary international equilibria.

THEOREM 1. *Assume that the two* ASSUMPTIONS e *and* E *are satisfied, and let* x(X) *be the unique autarchic stationary equilibrium in* ECONOMY 1(2). *Then:*

(1) *There exists a stationary international equilibrium with bundles* (x,X).

(2) *If* $|x_1 - e_1| > |X_1 - E_1|$, *then there exists a quasi-stationary equilibrium with bundles* $(\tilde{x},\tilde{X})$ *and a price system* $(\tilde{\pi}_t)_{t=1}^{\infty}$ *such that* $\tilde{\pi}_t/\tilde{\pi}_{t+1} = \tilde{\theta}_t = \tilde{\Theta} < 1$ *(an inflationary equilibrium).*

(3) *If* $|x_1 - e_1| < |X_1 - E_1|$, *then there exists a quasi-stationary equilibrium with bundles* $(\hat{x},\hat{X})$ *and a price system* $(\hat{\pi}_t)_{t=1}^{\infty}$ *such that* $\hat{\pi}_t/\hat{\pi}_{t+1} = \hat{\theta}_t = \hat{\Theta} > 1$ *(a deflationary equilibrium).*

(4) *Any quasi-stationary* (x',X') *is of the type described under* (1) - (3).

Proof:
Assertion (1) was established above. For (2), suppose that $|x_1 - e_1| > |X_1 - E_1|$, and define total excess demand $\zeta(\pi_t,\pi_{t+1})$ by

$$\zeta(\pi_t,\pi_{t+1}) = z(\pi_t,\pi_{t+1}) + Z(\pi_t,\pi_{t+1}).$$

Consider a variation in π_t for fixed $\pi_{t+1} = 1$. If $\pi_t = 1$, then $z(1,1) = x - e$, $Z(1,1) = X - E$, so by assumption $\zeta_1(1,1) < 0$. For π_t going to zero $Z_1(\pi_t,1)$ tends to $+\infty$ whereas $z_1(\pi_t,1)$ is bounded from below by $-e_1$. Consequently, $\zeta_1(\pi_t,t) > 0$ for π_t small enough. It follows from continuity that $\zeta_1(\tilde{\theta},1) = 0$ for some $\tilde{\theta} < 1$. Then $\tilde{x} = z(\tilde{\theta},1) + e$, $\tilde{X} = Z(\tilde{\Theta},1) + E$ are quasi-stationary equilibrium bundles with the price system $(\tilde{\pi}_t)_{t=1}^{\infty}$ given by $\tilde{\pi}_t = (1/\tilde{\theta})^t$.

Since $\tilde{\theta} \neq 1$ and $\tilde{\theta} z_1(\tilde{\theta},1) + z_2(\tilde{\theta},1) = 0$ (Walras Law), it follows that $z_1(\tilde{\theta},1) \neq -z_2(\tilde{\theta},1)$. Hence, $\tilde{x}_1 + \tilde{x}_2 \neq e_1 + e_2$ showing non-triviality of $(\tilde{x},\tilde{X})$.

The proof of assertion (3) is similar to that of (2) and is left to the reader. Finally, to establish (4), suppose that (x',X') is a quasi-stationary equilibrium with price system $(\pi'_t)_{t=1}^{\infty}$. If $x'_1 - e_1 \neq 0$, then

$$-\frac{x'_2 - e_2}{x'_1 - e_1} = \frac{\pi'_t}{\pi'_{t+1}} = \theta'$$

for any t, so $\theta'_t = \theta'$ is independent of t, and (x',X') must be either stationary $(\theta' = 1)$, inflationary $(\theta' < 1)$, or deflationary $(\theta' > 1)$. If $x'_1 - e_1 = 0$, then $-(x'_2 - e_2) = (\pi'_t/\pi'_{t+1})(x'_1 - e_1) = 0$. Hence, $x' = e$, and by ASSUMPTION e, $\pi'_t/\pi'_{t+1} < 1$ for all $t \geqq 1$. Feasibility (ii) implies

$$X'_1 - E_1 + X'_2 - E_2 = 0,$$

and Walras Law yields

$$(\pi'_t/\pi'_{t+1})(X'_1 - E_1) + (X'_2 - E_2) = 0$$

so $X' = E$. However, by ASSUMPTION E this means that $\pi'_t/\pi'_{t+1} > 1$ for all $t \geq 1$, a contradiction showing that $x'_1 - e_1 \neq 0$. □

Broadly speaking, the theorem says that apart from the no-trade equilibrium, where each country takes care itself of the necessary borrowing and lending, there is at least one additional quasi-stationary equilibrium. There borrowing between generations is abolished. Instead generation t in ECONOMY 1 lends the commodity at time t to the same generation in ECONOMY 2 and gets it back with (positive or negative) interest in period t+1. The quasi-equilibrium will be inflationary if under autarcy the supply of the good from the young (savers) in ECONOMY 1 exceeds the demand from the young (borrowers) in ECONOMY 2, and deflationary otherwise.

Assertion (1) of the theorem, i.e. that the autarchic equilibrium is also a stationary international equilibrium, contrasts markedly with classical trade theory. There international trade usually results in different, in certain cases even superior allocations. Here, however, this feature depends among other things on the simple structure of our model (cf. the remark in Section 2). With more than one good in each period, the autarchic equilibria would typically not be an international equilibrium.

The geometry of international equilibria is a straightforward exercise once the offer curves $d(z_t)$ and $D(Z_t)$ for the two countries are given, as illustrated already in Figures 1 and 2. Aggregate excess demand $z(\pi_t,\pi_{t+1}) + Z(\pi_t,\pi_{t+1})$ is found in Figure 3 by adding the two vectors obtained as intersection of d, resp D with the line $\{(z_1,z_2)\,|\,\pi_t z_1 + \pi_{t+1} z_2 = 0\}$. The aggregate offer curve denoted $d+D$ intersects the line $\{\lambda(-1,1)\,|\,\lambda \in R\}$ in B corresponding to the trivial stationary equilibrium. In the diagram it passes through the origin for some price ratio $\tilde{\theta} < 1$.

In our previous discussion, we have already touched upon the welfare aspects of opening up international trade. As will be seen presently, the situation in the infinite horizon model of international trade is strikingly different from the classical finite horizon case. In order to discuss welfare questions, some further terminology is needed. An *international allocation* is a pair $\left((x^t)_{t=0}^{\infty},(X^t)_{t=0}^{\infty}\right)$ where $x^t(X^t)$ is a bundle for gene-

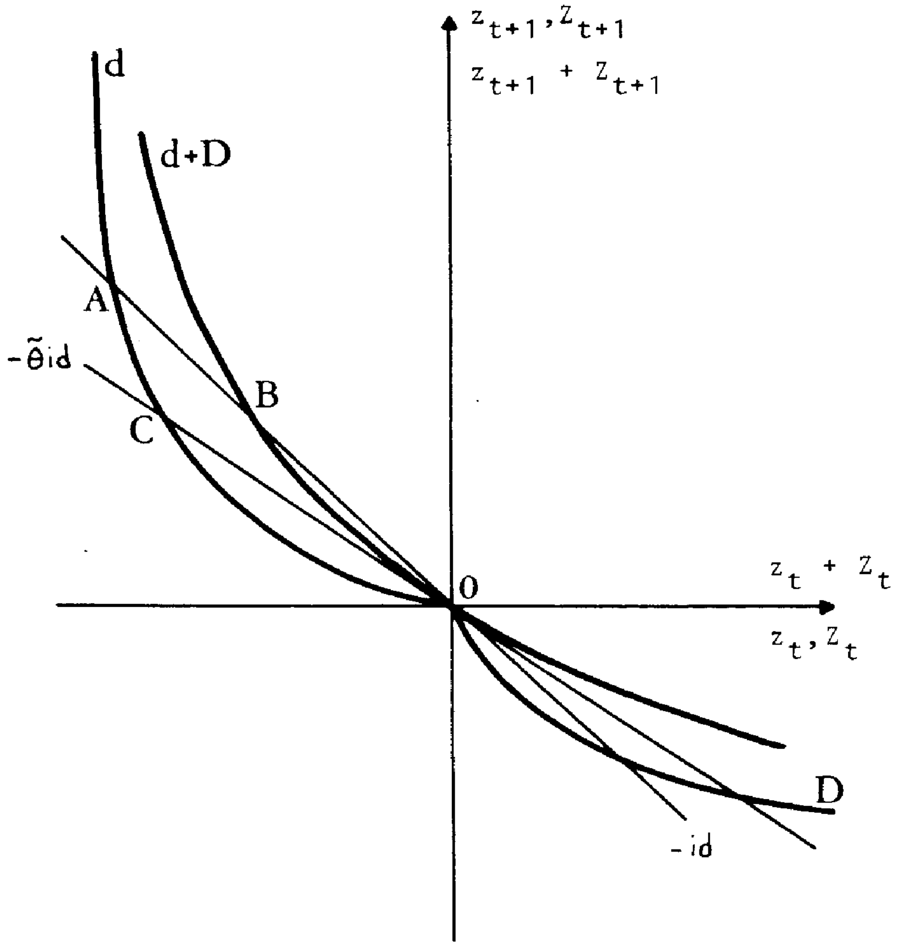

Figure 3

ration t in country 1(2). The international allocation is *feasible* if it satisfies (ii) above. A feasible allocation is *Pareto-efficient* if there is no feasible international allocation $\left((\bar{x}^t)_{t=0}^{\infty}, (\bar{X}^t)_{t=0}^{\infty}\right)$ such that

$$u_0(\bar{x}^0) \geqq u_0(x^0),\ U_0(\bar{X}^0) \geqq U_0(X^0),$$
$$u(\bar{x}^t) \geqq u(x^t),\ U(\bar{X}^t) \geqq U(X^t),\ t \geqq 1,$$

with strict inequality for some generation $t \geqq 0$ in some country.

THEOREM 2. *Let* $(\tilde{x},\tilde{X})$ *be a quasi-stationary equilibrium with price system* $(\tilde{\pi}_t)_{t=1}^{\infty}$; *such that* $\tilde{x} > 0$, $\tilde{X} > 0$, *and* $\tilde{\pi}_t/\tilde{\pi}_{t+1} = \tilde{\theta}_t$ *is independent of* t. *Then the equilibrium allocation is not Pareto-efficient if and only if* $\tilde{\theta}_t < 1$.

This theorem is an easy consequence of standard results on efficiency in infinite horizon models which obviates a demonstration (cf.e.g. Balasko and Shell (1980)). However, in our simple model a direct proof is straightforward and it may be instructive.

Proof:

Let $P(x)(P(X))$ be the set of bundles at least as good as $x(X)$ for the consumers of generation $t \geqq 1$ in ECONOMY 1(2), and let

$$S = P(x) - \{x\} + P(X) - \{X\}.$$

It is easily seen that the allocation $\left((\tilde{x}^t)_{t=1}^{\infty},(\tilde{X}^t)_{t=1}^{\infty}\right)$ with $\tilde{x}^t = \tilde{x}$, $\tilde{X}^t = \tilde{X}$, $t \geqq 1$, is inefficient if and only if there exists a sequence $(\varepsilon_t)_{t=1}^{\infty}$ with $\varepsilon_t \geqq 0$, all t, $\varepsilon_t > 0$, some t, and $(-\varepsilon_t,\varepsilon_{t+1}) \in S$ for all t.

Suppose that $\tilde{\pi}_t/\tilde{\pi}_{t+1} < 1$. Clearly, $(\tilde{\pi}_t,\tilde{\pi}_{t+1})$ supports the set S for each t. Consequently, there exists $\varepsilon > 0$ such that $(-\varepsilon,\varepsilon) \in S$, and the constant sequence $(\varepsilon,\varepsilon,\ldots)$ satisfies the above conditions.

If $\tilde{\pi}_t/\tilde{\pi}_{t+1} > 1$ and $(\varepsilon_t)_{t=1}^{\infty}$ is a sequence as above, we have $-\tilde{\pi}_t\varepsilon_t + \tilde{\pi}_{t+1}\varepsilon_{t+1} \geqq 0$ for all t. Hence,

$$\varepsilon_{t+1} \geqq \frac{\tilde{\pi}_t}{\tilde{\pi}_{t+1}}\,\varepsilon_t \geqq \left(\frac{\tilde{\pi}_t}{\tilde{\pi}_{t+1}}\right)^2 \varepsilon_{t-1} \geqq \left(\frac{\tilde{\pi}_t}{\tilde{\pi}_{t+1}}\right)^t \varepsilon_1.$$

Therefore, $\varepsilon_{t+1} \to \infty$, contradicting that $(-\varepsilon_{t+1},\varepsilon_{t+2}) \in S$ since S is bounded from below by $-e+E$.

If $\tilde{\pi}_t/\tilde{\pi}_{t+1} = 1$ and t is such that $\varepsilon_t > 0$, then $\varepsilon_{t+1} > \varepsilon_t + \delta_t$ for some $\delta_t > 0$. Thus, the sequence $(\varepsilon_t)_{t=1}^{\infty}$ is increasing. If it is bounded, it converges to some $\varepsilon_o > 0$, but then $(-\varepsilon_o, \varepsilon_o) \in S$ contradicting strict quasi-concavity of the utility functions. It follows that $(\varepsilon_t)_{t=1}^{\infty}$ is unbounded, so again we have a contradiction.

□

THEOREM 2 points to a fundamental difference between the welfare theory of international trade in the present set-up and the classical results. In our model, free trade may result in a Pareto-inefficient allocation. This, of course, does not mean that "trade decreases welfare" since the utility of one country's citizens is improved while that of the other country's is diminished. But the result, striking enough in itself, indicates that the traditional institutions for achieving Pareto-efficiency fails (or at least may fail in a considerable number of situations) when futures markets or capital movements are included in the model. The failure of free trade to assure intertemporal efficiency poses the problem of designing institutions which achieve efficiency. This is the topic of the following sections.

4. RATIONING EQUILIBRIA AND THE IMPOSSIBILITY OF IMPROVEMENT

In the previous section we found that in some situations international free trade may lead to Pareto inferior allocations. The notion of a Pareto improvement in overlapping generations models involves a change of bundles for infinitely many agents. In the proof of THEOREM 2 it was shown that a Pareto improvement could be enforced by direct intervention.

Since direct intervention in most cases is informationally costly and therefore usually not desirable, it is important to investigate, whether some Pareto improvement may be achieved by a proper institutional arrangement. This institution cannot be the market, of course, which produced inoptimality in the first place.

The particular institution of trade at non-Walrasian prices plus rationing almost suggests itself, since voluntary but restricted exchanges seem to be an institution which interferes as little as possible with the market mechanism while still producing a different set of equilibria. Moreover, in the context of a closed i.e. a one-country overlapping generations model, it was shown by Böhm and Puhakka (1985) that rationing equilibria can achieve improvements over competitive equilibria.

Defining an international equilibrium with quantity constraints we follow the standard concept as originally suggested by Drêze (1975). Quantity constraints for each agent $t = 0,1,\ldots$ take the form of upper and lower bounds on net trades, thus excluding forced trades for any agent. Hence, a list $(r^t,R^t)_{t=0}^{\infty}$ such that $r^0 = (\underline{r}_1^0,\overline{r}_1^0) \in \mathbb{R}^2$ with $\underline{r}_1^0 \leqq 0 \leqq \overline{r}_1^0$ ($R^0 = (\underline{R}_1^0,\overline{R}_1^0) \in \mathbb{R}^2$ with $\underline{R}_1^0 \leqq 0 \leqq \overline{R}_1^0$ respectively), and $r^t = (\underline{r}^t,\overline{r}^t) \in \mathbb{R}^4$ with $\underline{r}^t \leqq 0 \leqq \overline{r}^t$, $\underline{r}^t = (\underline{r}_t^t,\underline{r}_{t+1}^t)$, $\overline{r}^t = (\overline{r}_t^t,\overline{r}_{t+1}^t)$, $t \geqq 1$, ($R^t = (\underline{R}^t,\overline{R}^t) \in \mathbb{R}^4$ with $\underline{R}^t \leqq 0 \leqq \overline{R}^t$, $\underline{R}^t = (\underline{R}_t^t,\underline{R}_{t+1}^t)$, $\overline{R}^t = (\overline{R}_t^t,\overline{R}_{t+1}^t)$, $t \geqq 1$, respectively) defines a system of quantity constraints in the two economies.

A list $\left((x^t)_{t=0}^{\infty},(X^t)_{t=0}^{\infty}(r^t,R^t)_{t=0}^{\infty},(\pi_t)_{t=1}^{\infty},\omega\right)$ is an *international equilibrium with quantity constraints* if (i.a) $x^0(X^0)$ maximizes $u_o(U_o)$

subject to

$$\pi_1 x^0 \leqq \pi_1 e^0 + m, \ \underline{r}_1^0 \leqq x^0 - e^0 \leqq \overline{r}_1^0$$

$$(\pi_1 X^0 \leqq \pi_1 E^0 + \omega M, \ \underline{R}_1^0 \leqq X^0 - E^0 \leqq \overline{R}_1^0)$$

(i.b) for $t \geqq 1$:

$x^t(X^t)$ maximizes $u(U)$

subject to

$$\pi_t x_t^t + \pi_{t+1} x_{t+1}^t \leqq \pi_t e_t^t + \pi_{t+1}^t e_{t+1}^t$$

$$\underline{r}^t \leqq x^t - e^t \leqq \overline{r}^t$$

$$\left(\begin{array}{c} \pi_t X_t^t + \pi_{t+1} X_{t+1}^t \leqq \pi_t E_t^t + \pi_{t+1} E_{t+1}^t \\ \underline{R}^t \leqq X^t - E^t \leqq \overline{R}^t \end{array}\right)$$

(ii) for $t \geqq 1$:

$$x_t^{t-1} + X_t^{t-1} + x_t^t + X_t^t = e_t^{t-1} + E_t^{t-1} + e_t^t + E_t^t.$$

(iii) for $t \geqq 1$:

(a) if $x_t^\tau - e_t^\tau = \overline{r}_t^\tau$ or $X_t^\tau - E_t^\tau = \overline{R}_t^\tau$ some $\tau \in \{t,t-1\}$,

then

$x_t^\tau - e_t^\tau > \underline{r}_t^\tau$ and $X_t^\tau - E_t^\tau > \underline{R}_t^\tau$ all $\tau \in \{t,1-1\}$,

(b) if $x_t^\tau - e_t^\tau = \underline{r}_t^\tau$ or $X_t^\tau - E_t^\tau = \underline{R}_t^\tau$ some $\tau \in \{t,t-1\}$,

then

$x_t^\tau - e_t^\tau < \overline{r}_t^\tau$ and $X_t^\tau - E_t^\tau < \overline{R}_t^\tau$ all $\tau \in \{t,t-1\}$.

The definition of an equilibrium with quantity constraints in the context of overlapping generations may seem rather involved. However, it is a straightforward application of the definitional criteria given by Drèze (1975) to the present model. Comparing the definition with the one of an international competitive equilibrium in the previous section, we note that the main difference is due to the presence of the quantity restrictions (r^t, R^t) which restrict the set of feasible net trades of generations t-1 and t present at time t. Condition (iii) states that quantity constraints in the (single) market at each time t should not be binding on both the supply and the demand side simultaneously. This is the usual onesidedness condition. Hence, all equilibria with quantity constraints preserve the features of voluntary trading and of one sided rationing.

Some properties of the institutional framework of a rationing equilibrium may deserve some comments. Following the tradition of Drèze (1975), Malinvaud (1977) and Benassy (1975) we assume that there is a particular medium of exchange, money. Trade takes place in markets for the single good at any particular time t against money. Moreover, money serves as a store of value, so that generation t has access to the market for goods at time t+1. Since every generation $t \geqq 1$ has no initial money endowment, and money has no utility, it does not enter into the equilibrium conditions except for the initial generation.

The restrictions on net trades will have rather obvious interpretations in some cases. Suppose at market $t \geqq 1$ supply is rationed. This implies for generation t and t-1 that they cannot obtain as much money as desirable against delivery of commodities at the going prices. Hence, for generation t this is equivalent to demand rationing for money, and for generation t-1 this is equivalent to credit rationing in period t-1. Therefore, rationing on commodities in any period t implies restrictions on spot trades as well as on forward trades, with forward and backward spillover effects.

As is well known, there are at least two possible interpretations of equilibria in the overlapping generations model. First, they can be interpreted in the same way as is done when treating finite horizon models, i.e. that all trades are supposed to be contracted upon at $t = 1$. This has the disadvantage of involving all the unborn generations in the contracting process (which of course is outside the model). The other interpretation is more suitable in our context. Here prices and quantities are the outcome of a rational (or rather correct) expectations equilibrium in the following sense. Consumers trade at time t only in the market for the good at that time (against money), having certain expectations with respect to future prices and rationings. In the equilibrium, the expectations must be correct and, moreover, price expectations should be the same for all agents. This being so, there will be no need for a distinction between spot and forward trading for the good at time t.

Suppose now that the two-country model is such that case (2) of THEOREM 1 applies, so that there is an inflationary quasi-stationary equilibrium with bundles (x,X) and price system $(\pi_t)_{t=1}^{\infty}, \pi_t/\pi_{t+1} = \theta < 1$. One knows from THEOREM 2 that the equilibrium allocation can be Pareto improved. THEOREM 3 provides an answer to the more interesting question of whether it can be improved through rationing, that is to say, whether there exists an international equilibrium with quantity constraints relative to some price system $(\hat{\pi}_t)_{t=1}^{\infty}$ such that its allocation is Pareto superior to (x,X).

THEOREM 3. *Let* $\left((x^0,X^0),(x,X),(\pi_t)_{t=1}^{\infty}\right)$ *be a quasi-stationary international equilibrium such that* $x \neq e, X \neq E$, *(each country and generation has a non-zero net trade). If* ASSUMPTION E *holds, then there is no international equilibrium with quantity constraints which Pareto dominates* $\left((x^0,X^0),(x,X)\right)$.

The proof of THEOREM 3 given below is rather long. The reader not interested in technical details may proceed directly to the graphical illustration following the proof.

Proof:

First recall that at the inflationary quasi-stationary equilibrium (x,X) with $\theta < 1$ intergenerational trade is eliminated, i.e.

(i) $x^0 - e^0 + X^0 - E^0 = 0$

(ii) $x_1 - e_1 + X_1 - E_1 = 0 = x_2 - e_2 + X_2 - E_2$.

This follows immediately from feasibility

$$x_1 - e_1 + X_1 - E_1 + x_2 - e_2 + X_2 - E_2 = 0$$

and from budget balance of each young generation with $\theta < 1$.

$$\theta(x_1 - e_1) + (x_2 - e_2) = 0$$
$$\theta(X_1 - E_1) + (X_2 - E_2) = 0.$$

Suppose there exists an international equilibrium with quantity constraints with prices $(\hat{\pi})_{t=1}^{\infty}$ and such that its allocation $\left((\hat{x}^t)_{t=0}^{\infty},(\hat{X}^t)_{t=0}^{\infty}\right)$ Pareto dominates $\left((x^0,X^0)(x,X)\right)$. There must be some generation $t \geqq 0$ such that either $\hat{x}^t \neq x$ or $\hat{X}^t \neq X$. If $\hat{x}^t_t = e_t$ or $\hat{X}^t_t = E_t$, then budget balance implies $\hat{x}^t_{t+1} = e_{t+1}$, or $\hat{X}^t_{t+1} = E_{t+1}$, and therefore $\hat{x}^t = e$ or $\hat{X}^t = E$. But $u(e) < u(x)$ and $U(E) < U(X)$ contradicting the fact that $\left((\hat{x}^t)(\hat{X}^t)\right)$ Pareto dominates (x,X).

Since x is the unique maximizer of u at prices θ, $u(\hat{x}^t) \geqq u(x)$ implies

$$\text{(iii)} \quad \theta \hat{x}^t_t + \hat{x}^t_{t+1} \geqq \theta e^t_t + e^t_{t+1}$$

with strict inequality if $\hat{x}^t \neq x$.

Let $\hat{\theta}_t = \hat{\pi}_t/\hat{\pi}_{t+1}$ and $\hat{x}^t \neq x$. Then, (iii) and the budget equation

$$\hat{\theta}_t \hat{x}^t_t + \hat{x}^t_{t+1} = \hat{\theta}_t e^t_t + e^t_{t+1}$$

imply

$$\text{(iv)} \quad (\hat{x}^t_t - e^t_t)(\theta - \hat{\theta}_t) > 0.$$

For generation t of ECONOMY 2, the counterpart of (iii) and the budget equation yields $(\hat{X}^t_t - E^t_t)(\theta - \hat{\theta}_t) \geqq 0$. Hence,

$$\text{(v)} \quad \begin{aligned} \hat{x}^t_t - e^t_t > 0 &\text{ iff } \hat{X}^t_t - E^t_t > 0 \\ \hat{x}^t_t - e^t_t < 0 &\text{ iff } \hat{X}^t_t - E^t_t < 0. \end{aligned}$$

In other words, for any Pareto improving sequence $\Big((\hat{x}^t),(\hat{X}^t)\Big)$, young agents in both economies are (strictly) on the same side of the market. Moreover, market balance in period t+1

$$\hat{x}^t_{t+1} - e^t_{t+1} + \hat{X}^t_{t+1} - E^t_{t+1} + \hat{x}^{t+1}_{t+1} - e^{t+1}_{t+1} + \hat{X}^{t+1}_{t+1} - E^{t+1}_{t+1} = 0$$

implies

$$\text{(vi)} \quad \begin{aligned} \hat{x}^{t+1}_{t+1} - e^{t+1}_{t+1} > 0 &\text{ iff } \hat{x}^t_t - e^t_t > 0 \\ \hat{x}^{t+1}_{t+1} - e^{t+1}_{t+1} < 0 &\text{ iff } \hat{x}^t_t - e^t_t < 0. \end{aligned}$$

Let $t \geqq 1$ denote the first generation for which $(\hat{x}^t,\hat{X}^t) \neq (x,X)$ and assume $\hat{x}^t_t - e^t_t > 0$ and $\hat{X}^t_t - E^t_t > 0$. If $t = 1$, then feasibility and condition (i) imply

$$\hat{x}^0 - e^0 + \hat{X}^0 - E^0 < 0.$$

Therefore, at least one of the initial consumers must be worse off. If $t > 1$, then feasibility and (ii) imply

$$\begin{aligned} 0 &= x_2 - e_2 + X_2 - E_2 + \hat{x}_t^t - e_t^t + \hat{X}_t^t - E_t^t \\ &= \hat{x}_t^t - e_t^t + \hat{X}_t^t - E_t^t > 0, \end{aligned}$$

which is a contradiction.

The remaining case is $\hat{x}_t^t - e_t^t < 0$ and $\hat{X}_t^t - E_t^t < 0$ from some $t \geqq 1$ on. ASSUMPTION E implies that $\hat{\theta}_t = \hat{\pi}_t/\hat{\pi}_{t+1} > 1$. Let $K > 1$ be defined by $K = U_2'(E)/U_1'(E)$. Budget balance yields

$$\hat{x}_{t+1}^t - e_{t+1}^t + \hat{X}_{t+1}^t - E_{t+1}^t = -\hat{\theta}\left(\hat{x}_t^t - e_t^t + \hat{X}_t^t - E_t^t\right)$$

and

$$\hat{x}_{t+1}^t - e_{t+1}^t + \hat{X}_{t+1}^t - E_{t+1}^t \geqq -K\left(\hat{x}_t^t - e_t^t + \hat{X}_t^t - E_t^t\right).$$

Feasibility implies

$$\hat{x}_{t+1}^{t+1} - e_{t+1}^{t+1} + \hat{X}_{t+1}^{t+1} - E_{t+1}^{t+1} \leqq K\left(\hat{x}_t^t - e_t^t + \hat{X}_t^t - E_t^t\right).$$

Using (vi) and repeating the argument successively yields after n steps

$$\hat{x}_{t+n}^{t+n} - e_{t+n}^{t+n} + \hat{X}_{t+n}^{t+n} - E_{t+n}^{t+n} \leqq K^n\left(\hat{x}_t^t - e_t^t + \hat{X}_t^t - E_t^t\right).$$

For $n \to \infty$ total net supply and therefore the net supply of at least one country are unbounded contradicting that consumption sets are bounded from below and endowments of each generation are bounded.

□

It should be noted that nowhere in the proof there is an argument using quantity constraints. Therefore the result includes as a special case the following corollary.

COROLLARY. *If* ASSUMPTION E *holds and if each country and each generation has a non-zero net trade at an inflationary quasi-stationary international trade equilibrium, then no other (non-stationary) international equilibrium can Pareto improve the quasi-stationary equilibrium.*

To give an intuitive explanation of the result we return to the well-behaved special case developed in the earlier sections. For simplicity, we assume that $e = (e_1, 0)$, $E = (0, E_2)$. This means that in the diagram of Figure 4, which essentially

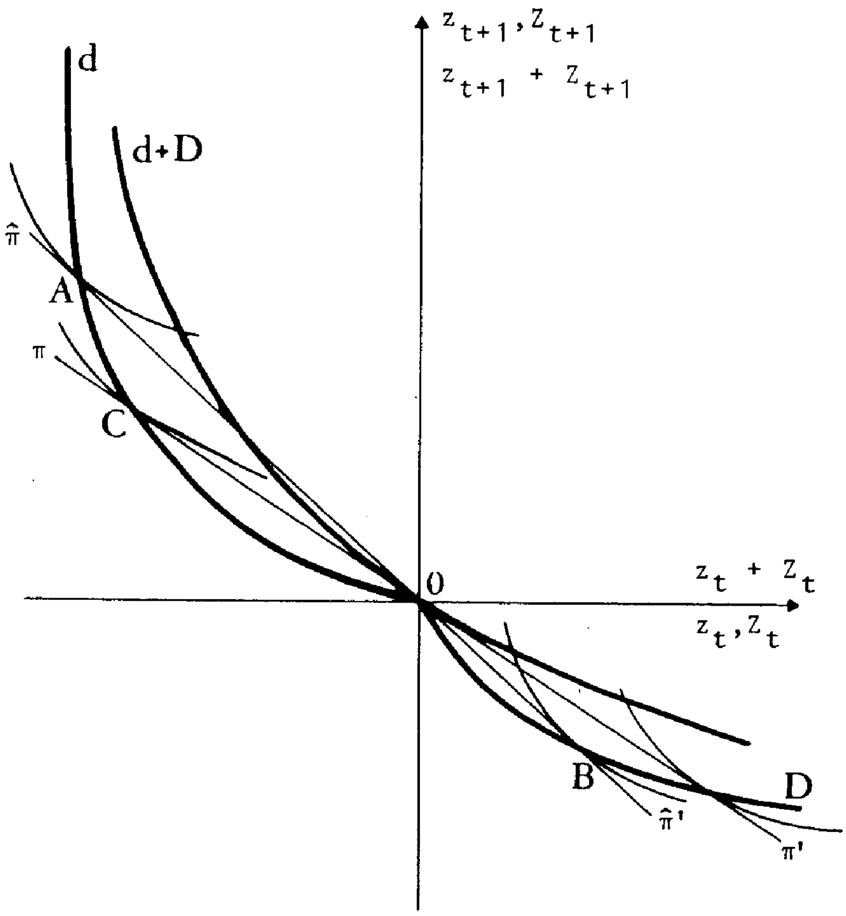

Figure 4

reproduces that of Figure 3, the net trades of consumers in the first country must be in the second quadrant, those of the consumers in country 2 in the fourth quadrant.

In the figure, the line $\pi\pi'$ is the price line of the quasi-stationary equilibrium, whereas $\hat{\pi}\hat{\pi}'$ is the one belonging to the equilibrium with quantity constraints (at time t,t+1). Clearly, the price lines cannot coincide, since then the bundles of the original equilibrium would be the unique utility maximizers. If price lines differ, then the new net trade of one of the consumers must be below the original price line, and therefore inferior to the original net trade, or it must be 0. But the latter case is already excluded, since the original bundle was better than the initial endowment.

5. INTERNATIONAL EQUILIBRIA WITH PRICE DISPARITIES

The results of the preceding section indicate that the Pareto inefficiency of inflationary competitive equilibria cannot be remedied through markets while maintaining identical intertemporal prices in the two countries, even when recourse is taken to quantity restrictions. In view of the Corollary, the result also implies that a constant exchange rate and budget balance for each generation in each country make a Pareto improvement impossible, even if forced trading were allowed. As a consequence, situations have to be considered where there is no common world price or, which is equivalent, where the exchange rate is not constant over time.

Price distortions among countries may take the form of either different relative prices at any particular date on the associated spot markets or of different intertemporal prices, i.e. of differing exchange rates. The first type of situation arises in a fairly natural way in models with tariffs or export subsidies. In the present model with trade in commodities delivered at different dates, the interpretation of price differences being caused by an imposition of tariffs may not be convincing. Hence, the second type of price disparity is the more appropriate one to be considered here. The typical economic institution in such a case would be a government monopoly of trade in foreign currencies and/or an active exchange rate policy.

The possibility of price disparities among the two countries necessitates that the two national price systems have to be made explicit in the definition of the associated equilibrium concept. The resulting exchange rate will then be defined in the usual way.

An *international equilibrium with price disparities* is a quadruple $\left((x^t)_{t=0}^{\infty},(p_t)_{t=1}^{\infty},(X^t)_{t=0}^{\infty},(P_t)_{t=1}^{\infty}\right)$ such that

(i.a) $x^0(X^0)$ maximizes $u_0(U_0)$ subject to

$$p_1 x^0 \leqq p_1 e^0 + m$$

$$(P_1 X^0 \leqq P_1 E^0 + M)$$

(i.b) for $t \geqq 1$:

$x^t(X^t)$ maximizes $u(U)$ subject to

$$p_t x_t^t + p_{t+1} x_{t+1}^t \leqq p_t e_t^t + p_{t+1} e_{t+1}^t$$

$$(P_t X_t^t + P_{t+1} X_{t+1}^t \leqq P_t E_t^t + P_{t+1} E_{t+1}^t)$$

(ii) for $t \geqq 1$:

$$x_t^{t-1} + X_t^{t-1} + x_t^t + X_t^t = e_t^{t-1} + E_t^{t-1} + e_t^t + E_t^t .$$

The concept of an international competitive equilibrium introduced in section 3 clearly is a special case of an equilibrium with price disparities. As before, an equilibrium will be called *quasi-stationary* if $x^t = x^{t'}$ and $X^t = X^{t'}$ for all $t,t' \geqq 1$. There is no need to consider stationary equilibria with price disparities separately, since they do not differ from stationary competitive equilibria.

The goal of this section is to model institutions which may sustain a Pareto improvement over an inflationary international competitive equilibrium. PROPOSITION 3 supplies a positive answer using country specific prices. The result depends, however, on some additional assumptions on consumer characteristics.

PROPOSITION 3: *Let* $\left((\tilde{x}^0,\tilde{X}^0),(\tilde{x},\tilde{X});\tilde{\theta}\right)$ *denote an inflationary quasi-stationary competitive international equilibrium such that* $\tilde{x} \neq e$ *and* $\tilde{X} \neq E$. *If the conditions*

(e*) $\tilde{x}_1 - e_1 < 0$ *and* *for some* $p_t/p_{t+1} > \tilde{\theta}$ *there exists a demand bundle* x *for agents in* ECONOMY 1 *with* $\tilde{x}_1 + \tilde{x}_2 \geqq x_1 + x_2$, *or*

(E*) $\tilde{X}_1 - E_1 > 0$ *and* *for some* $P_t/P_{t+1} < \tilde{\theta}$ *there exists a demand bundle* X *for agents in* ECONOMY 2 *with* $\tilde{X}_1 + \tilde{X}_2 \geqq X_1 + X_2$ *hold,*

then there exists an international equilibrium with price disparities $\left((x^t)_{t=0}^{\infty},(p_t)_{t=1}^{\infty},(X^t)_{t=0}^{\infty},(P_t)_{t=1}^{\infty}\right)$ *which Pareto dominates* $\left((x^0,X^0),(\tilde{x},\tilde{X})\right)$. *Moreover, the Pareto dominating equilibrium can be chosen to be quasi-stationary with inflation rates such that* $1 > \theta = p_t/p_{t+1} \geqq \tilde{\theta} \geqq P_t/P_{t+1} = \Theta > 0$, *where one of the inequalities is strict.*

Conditions (e*) and (E^*) actually state properties of the demand behavior of the exporting country (e*) or of the importing country (E^*). Here, it is assumed that ECONOMY 1 is the net exporting country. This represents no loss of generality. The proof will be carried out for the case that (e*) holds.

Proof:

For each $\theta > \tilde{\theta}$ let $g(\theta) = x_1 + x_2$ where $(x_1, x_2) = \arg\max \left\{u(x_1,x_2) \mid \theta x_1 + x_2 \leqq \theta e_1 + e_2\right\}$. Clearly, g is continuous and $g(\theta) \to \infty$ as θ tends to infinity. (e*) implies that there exists $\bar{\theta} > \tilde{\theta}$ such that

$u(\bar{x}) = \text{Max}\left\{u(x) \,\middle|\, \bar{\theta}x_1 + x_2 \leqq \bar{\theta}e_1 + e_2\right\}$ with $\bar{x}_1 + \bar{x}_2 = g(\bar{\theta}) = \tilde{x}_1 + \tilde{x}_2$. Hence, $\bar{\theta}\tilde{x}_1 + \tilde{x}_2 = \bar{\theta}\tilde{x}_1 + e_2 - \tilde{\theta}(\tilde{x}_1 - e_1)$ and $\tilde{x}_1 - e_1 < 0$ implies $\bar{\theta}\tilde{x}_1 + \tilde{x}_2 < \bar{\theta}e_1 + e_2$, so that $u(\bar{x}) > u(\tilde{x})$ and $\tilde{\theta}(\bar{x}_1 - e_1) + (\bar{x}_2 - e_2) > 0$. The budget identity then yields $\bar{x}_1 - e_1 < 0$ and

$$0 = \bar{\theta}(\bar{x}_1 - e_1) + (\bar{x}_2 - e_2) = (\bar{x}_1 - e_1)(\bar{\theta} - 1) + \tilde{x}_1 - e_1 + \tilde{x}_2 - e_2$$
$$< (\bar{x}_1 - e_1)(\bar{\theta} - 1) + \tilde{\theta}(\tilde{x}_1 - e_1) + (\tilde{x}_2 - e_2) = (\bar{x}_1 - e_1)(\bar{\theta} - 1).$$

Therefore, $0 < \tilde{\theta} < \bar{\theta} < 1$. Finally, $\bar{\theta}\tilde{x}_1 + \tilde{x}_2 < \bar{\theta}\bar{x}_1 + \bar{x}_2$ yields $(\bar{\theta} - 1)(\tilde{x}_1 - \bar{x}_1) < 0$, which implies $\tilde{x}_1 > \bar{x}_1$. Hence, there exists $\bar{\lambda} > 0$ such that $\bar{x} = \tilde{x} + \bar{\lambda}(-1,1)$.

Let $k = m/(\tilde{x}^0 - e^0 + \bar{\lambda})$ and define the price sequence $(p_t)_{t=1}^{\infty}$ for ECONOMY 1 by $p_t = k\bar{\theta}^{1-t}$, $t \geqq 1$. By construction, one has $p_t/p_{t+1} = \bar{\theta}$ and, for each generation $t \geqq 1$,

$$z(p_t, p_{t+1}) = (\tilde{x} - e) + \bar{\lambda}(-1,1).$$

Therefore, the allocation $\left((\bar{x}^t)(\bar{X}^t)\right)$ such that $\bar{x}^0 = \tilde{x}^0 + \bar{\lambda}$, $\bar{X}^0 = \tilde{X}^0$ and, $\bar{x}^t = \bar{x}$ and $\bar{X}^t = \tilde{X}$ for $t \geqq 1$ Pareto dominates $(\tilde{x}, \tilde{X})$. Moreover, for each $t \geqq 1, (\bar{x}^t)(\bar{X}^t)$ is feasible since

$$\bar{x}_t^{t-1} + \tilde{X}_t^{t-1} + \bar{x}_t^t + \tilde{X}_t^t = \tilde{x}_2 + \bar{\lambda} + \tilde{X}_2 + \tilde{x}_1 - \bar{\lambda} + \tilde{X}_1$$
$$= e_2 + E_2 + e_1 + E_1.$$

Define the price sequence $(P_t)_{t=1}^{\infty}$ for ECONOMY 2 by

$$P_t = K\tilde{\theta}^{1-t} \quad \text{where} \quad K = -\frac{M}{\tilde{X}^0 - E^0} .$$

It follows now, that $\left((\bar{x}^t)_{t=0}^{\infty}, (p_t)_{t=1}^{\infty}, (\bar{X}^t)_{t=0}^{\infty}, (P_t)_{t=1}^{\infty}\right)$ is a quasi-stationary international equilibrium with price disparities which Pareto dominates $(\tilde{x}, \tilde{X})$. □

The proof of the PROPOSITION is completely symmetric to the one given above if (E*) holds instead of (e*). Then, agents in ECONOMY 2 will be made better off. It follows from the construction in the proof that all agents can be made better off under price disparities if both conditions (e*) and (E*) hold simultaneously.

The role of the two ASSUMPTIONS (e*) and (E*) can be illustrated in a diagrammatic representation. Assume as before that $E_1 = e_2 = 0$. Then young agents in ECONOMY 1 are always net suppliers and the condition $\tilde{x}_1 - e_1 < 0$ holds at the quasi-stationary competitive international equilibrium. For $\tilde{\theta} < 1$, condition (e*) implies that for some $\theta > \tilde{\theta}$, i.e. with less inflation, each generation's additional supply when young requires at most the same additional amount in demand when old for the utility maximizing consumption bundle associated with the lower inflation rate $1/\theta$. This translates immediately into the condition on the slope of the offer curve being less than one in absolute value at the inflationary equilibrium (see Figure 5). Hence, for some $\theta > \tilde{\theta}$, the offer curve d stays below the line $\lambda(-1,1) + \tilde{z}$ which has slope minus one. The intersection of

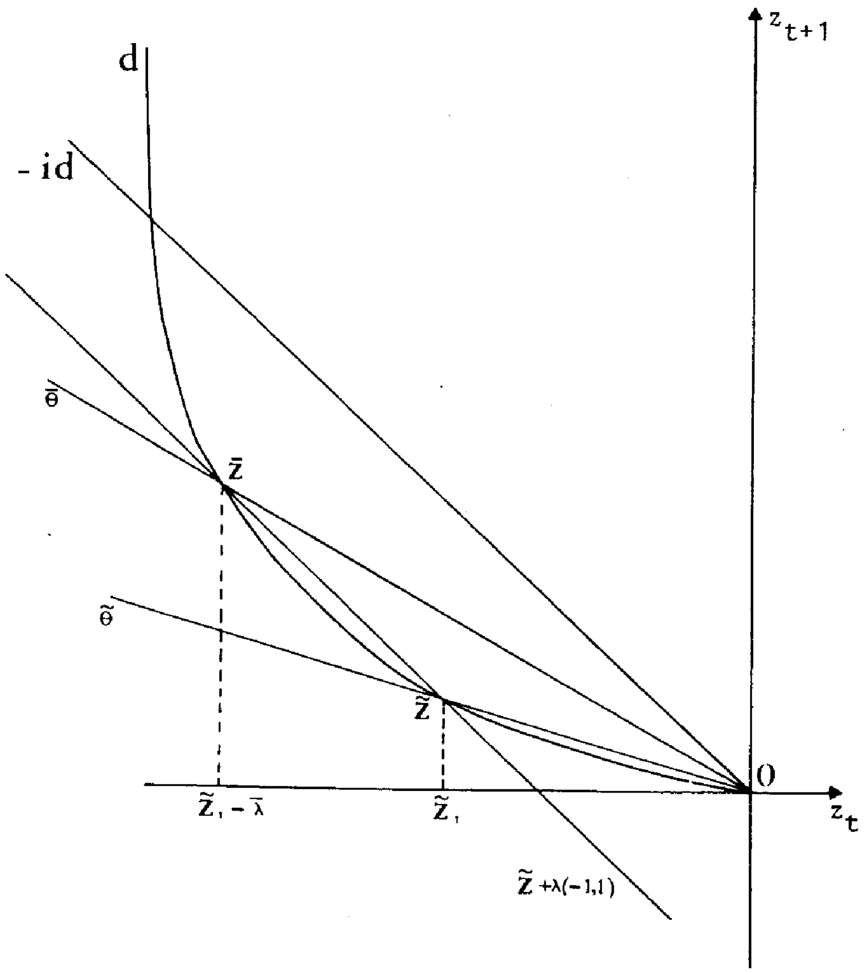

Figure 5

this line with the offer curve d defines the new inflation rate $1/\bar{\theta}$ and the new allocation $\bar{z}$ which has higher utility than $\tilde{z}$. It is immediate that there is renewed intergenerational trade of size $\bar{\lambda}$ after the inflation rate has been corrected. Figure 6 contains the equivalent geometric construction for ECONOMY 2 if (E*) holds. The assumption requires that D must be backward bending. $\bar{Z}$ is the Pareto improving demand point at the inflation rate $1/\Theta$ which is greater than $1/\tilde{\theta}$. $\bar{\lambda}$ denotes again the amount of intergenerational trade.

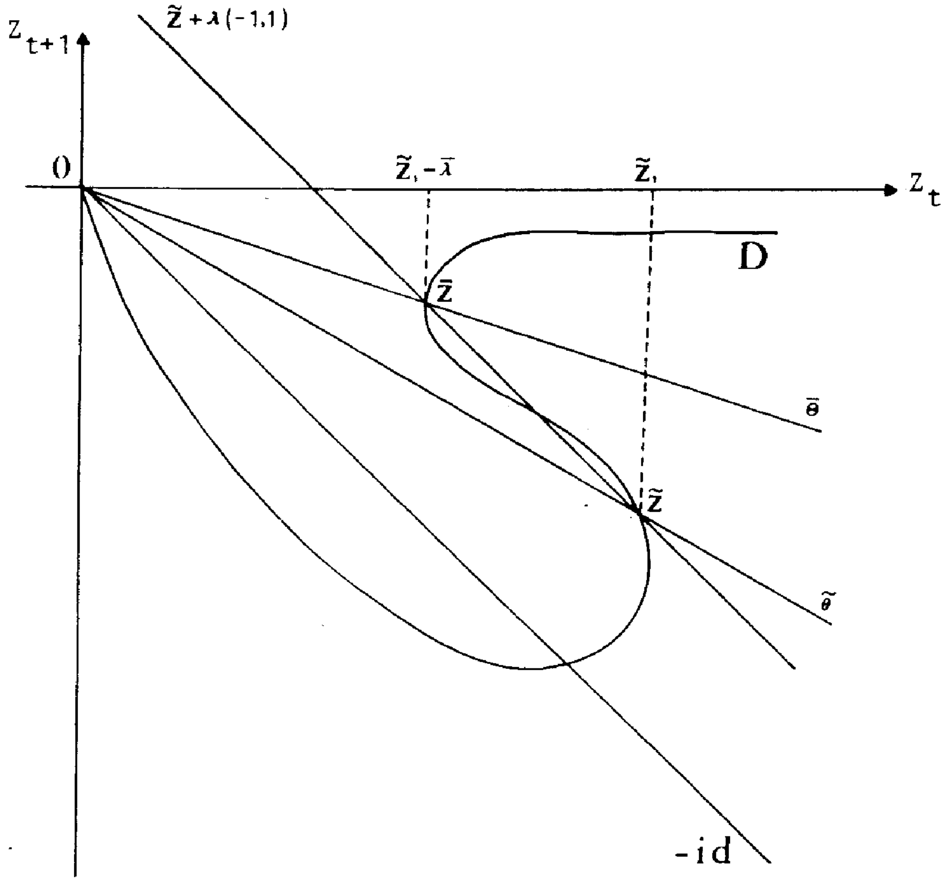

Figure 6

The assumptions (e*) and (E*) used in PROPOSITION 3 are not very useful since they state conditions on derived concepts such as demand functions at the inflationary equilibrium rather than on the basic characteristics of the economy, like e.g. preferences or endowments. The geometrical arguments illustrate quite strikingly what is at stake. Since, in general, utility maximization implies almost no restrictions on the shape of individual offer curves, a result on Pareto improving policies or institutions would be of little interest if some specific form of the offer curve would be required. Moreover, any reader should have no difficulty to draw any two well behaved offer curves in Figures 5 and 6 which violate (e*) as well as (E*), and thus making PROPOSITION 3 inapplicable. The final result of the paper, therefore, readdresses the question of quantity constraint equilibria under price disparities. At this stage no new ideas are involved. Quantity constraints as introduced in Section 4 of the paper are added to the equilibrium conditions given at the beginning of this section.

A quadruple $\left((x^t)_{t=0}^{\infty},(p_t)_{t=1}^{\infty},(X^t)_{t=0}^{\infty},(P_t)_{t=1}^{\infty}\right)$ together with a system of quantity constraints $(r^t,R^t)_{t=0}^{\infty}$ is called an *international equilibrium with price disparities and quantity constraints* if

(i.a) $x^0(X^0)$ maximizes $u_0(U_0)$ subject to

$$p_1x^0 \leqq p_1e^0 + m, \quad \underline{r}_1^0 \leqq x^0 - e^0 \leqq \overline{r}_1^0$$

$$(P_1X^0 \leqq P_1E^0 + M, \quad \underline{R}_1^0 \leqq X^0 - E^0 \leqq \overline{R}_1^0)$$

(i.b) for $t \geqq 1$:

$x^t(X^t)$ maximizes $u(U)$ subject to

$$p_tx_t^t + p_{t+1}x_{t+1}^t \leqq p_te_t^t + p_{t+1}e_{t+1}^t$$

$$\underline{r}^t \leqq x^t - e^t \leqq \overline{r}^t$$

$$P_tX_t^t + P_{t+1}X_{t+1}^t \leqq P_tE_t^t + P_{t+1}E_{t+1}^t$$

$$\underline{R}^t \leqq X^t - E^t \leqq \overline{R}^t$$

(ii) for $t \geqq 1$:

$$x_t^{t-1} + X_t^{t-1} + x_t^t + X_t^t = e_t^{t-1} + E_t^{t-1} + e_t^t + E_t^t$$

(iii) for $t \geq 1$:

(a) if $x_t^\tau - e_t^\tau = \overline{r}_t^\tau$ or $X_t^\tau - E_t^\tau = \overline{R}_t^\tau$, some $\tau \in \{t,t-1\}$ then

$$x_t^\tau - e_t^\tau > \underline{r}_t^\tau \text{ and } X_t^\tau - E_t^\tau > \underline{R}_t^\tau, \text{ all } \tau \in \{t,t-1\}$$

(b) if $x_t^\tau - e_t^\tau = \underline{r}_t^\tau$ or $X_t^\tau - E_t^\tau = \underline{R}_t^\tau$, some $\tau \in \{t,t-1\}$ then

$$x_t^\tau - e_t^\tau < \overline{r}_t^\tau \text{ and } X_t^\tau - E_t^\tau < \overline{R}_t^\tau, \text{ all } \tau \in \{t,t-1\}.$$

THEOREM 4 provides the general affirmative answer that non-trivial quasi-stationary inflationary competetive equilibria can be improved upon using price disparities and rationing. It should be noted that the only assumption exploited in the proof is that the utility functions are differentiable. This assumption is not essential but reduces the number of steps required in the proof substantially. Strict quasi-concavity and monotonicity of preferences are sufficient to prove the result.

THEOREM 4: *Let* $\left((\tilde{x}^0,\tilde{x}),(\tilde{X}^0,\tilde{X}),\tilde{\theta}\right)$ *denote an inflationary quasi-stationary international competitive equilibrium such that* $\tilde{x} \neq e$ *and* $\tilde{X} \neq E$. *Then there exists a quasi-stationary international equilibrium with price disparities and quantity constraints, which Pareto dominates* $\left((\tilde{x}^0,\tilde{x}),(\tilde{X}^0,\tilde{X})\right)$.

Proof:
Let $\tilde{\theta} < 1$ and assume without loss of generality that $\tilde{X}_1 - E_1 > 0$. Since preferences are smooth, there exists $\overline{\lambda} > 0$ such that $U\left(\tilde{X} + \lambda(-1,1)\right) > U(\tilde{X})$ for all $\lambda, \overline{\lambda} > \lambda > 0$. Moreover, any such allocation for all $t \geqq 1$ and $\tilde{X}^0 + \lambda$ with $(x^0,\tilde{x})$ for ECONOMY 1 is feasible since $\tilde{x}_1 + \tilde{x}_2 + \tilde{X}_1 - \lambda + \tilde{X}_2 + \lambda = e_1 + e_2 + E_1 + E_2$. It remains to be shown that, for some λ small, such an allocation can be sustained as an equilibrium.

Let $X(\Theta) = \arg\max \left\{U(X) \,\middle|\, \Theta X_1 + X_2 \leqq \Theta E_1 + E_2\right\}$ for $\Theta \leqq \tilde{\theta}$. For $\Theta < \tilde{\theta}$, one obtains $X_1(\Theta) - E_1 > 0$, and therefore,

$$\left(X_1(\Theta) - E_1\right) + X_2(\Theta) - E_2 > \tilde{\theta}\left[X_1(\Theta) - E_1\right] + X_2(\Theta) - E_2$$
$$> \Theta\left[X_1(\Theta) - E_1\right] + X_2(\Theta) - E_2 = 0.$$

Hence, $U\left(X(\Theta)\right) > U(\tilde{X})$. If $X_1(\Theta) - E_1 + X_2(\Theta) - E_2 \leq \tilde{X}_1 - E_1 + \tilde{X}_2 - E_2$ for some $\Theta < \tilde{\theta}$, then condition (E*) holds and PROPOSITION 3 can be applied.

Suppose $X_1(\Theta) - E_1 + X_2(\Theta) - E_2 > \tilde{X}_1 - E_1 + \tilde{X}_2 - E_2$ for all $\Theta < \tilde{\theta}$. Rationing in the one commodity two period case takes a very simple form. It is easy to verify that an agent of ECONOMY 2 is demand constrained at the level $\bar{R}_1$ when young if and only if there exists $0 < \gamma < 1$ such that $\gamma\left(X_1(\Theta) - E_1\right) = \bar{R}_1$. Hence, it is sufficient to prove that for some $\Theta < \tilde{\theta}$ there exists $0 < \gamma < 1$ and $\lambda < \bar{\lambda}$ such that

$$\tilde{X} + \lambda(-1,1) = \gamma\left[X(\Theta) - E\right] + E,$$

or equivalently in vector notation,

$$\begin{bmatrix} X_1(\Theta) - E_1 & 1 \\ X_2(\Theta) - E_2 & -1 \end{bmatrix}\begin{bmatrix} \gamma \\ \lambda \end{bmatrix} = \begin{bmatrix} \tilde{X}_1 - E_1 \\ \tilde{X}_2 - E_2 \end{bmatrix}.$$

Since $X(\Theta)$ is a continuous function and since the matrix is non-singular, this yields a continuous function which maps Θ into $[0,1]\times[0,1]$. It is straightforward to check that $0 < \gamma(\Theta) < 1$ and $0 < \lambda(\Theta)$ for $\Theta < \tilde{\theta}$. Moreover, $\gamma(\Theta) \to 1$ and $\lambda(\Theta) \to 0$ as $\Theta \to \tilde{\theta}$. Hence, $\exists\, \bar{\Theta} < \tilde{\theta}$ such that

$$\bar{X} = \arg\max \left\{U(X) \,\middle|\, \bar{\Theta}X_1 + X_2 \leq \bar{\Theta}E_1 + E_2, \gamma(\bar{\Theta})(X_1 - E_1) \leq \tilde{X}_1 + E_1 - \lambda(\bar{\Theta})\right\}$$
$$= \gamma(\bar{\Theta})\left[X(\bar{\Theta}) - E\right] + E,$$

and such that $\lambda(\bar{\Theta}) < \bar{\lambda}$ and $\lambda(\bar{\Theta}) < E^0 - \tilde{X}^0$.

Define $(p_t)_{t=1}^{\infty}$ by

$$p_t = k\tilde{\theta}^{1-t} \quad \text{with} \quad k = m/(\tilde{x}^0 - e^0)$$

and

$$P_t = K\bar{\Theta}^{1-t} \quad \text{with} \quad K = M/(E^0 - \tilde{X}^0 + \lambda(\bar{\Theta})).$$

Then, $\left((\tilde{x}^0,\tilde{x})(p_t)_{t=1},(\tilde{X}^0 + \lambda(\bar{\Theta}),\bar{X})(P_t)_{t=1}^{\infty}\right)$ is a quasi-stationary equilibrium with price disparities and quantity rationing which Pareto dominates $(\tilde{x}^0,\tilde{x})(\tilde{X}^0,\tilde{X})$.

□

Figure 7 provides a geometric characterization. Since ECONOMY 1 is not involved in the Pareto improvement, only the net trade situation of agents in ECONOMY 2 are given. At the initial situation $\tilde{Z}$ the indifference curve tangent to the line

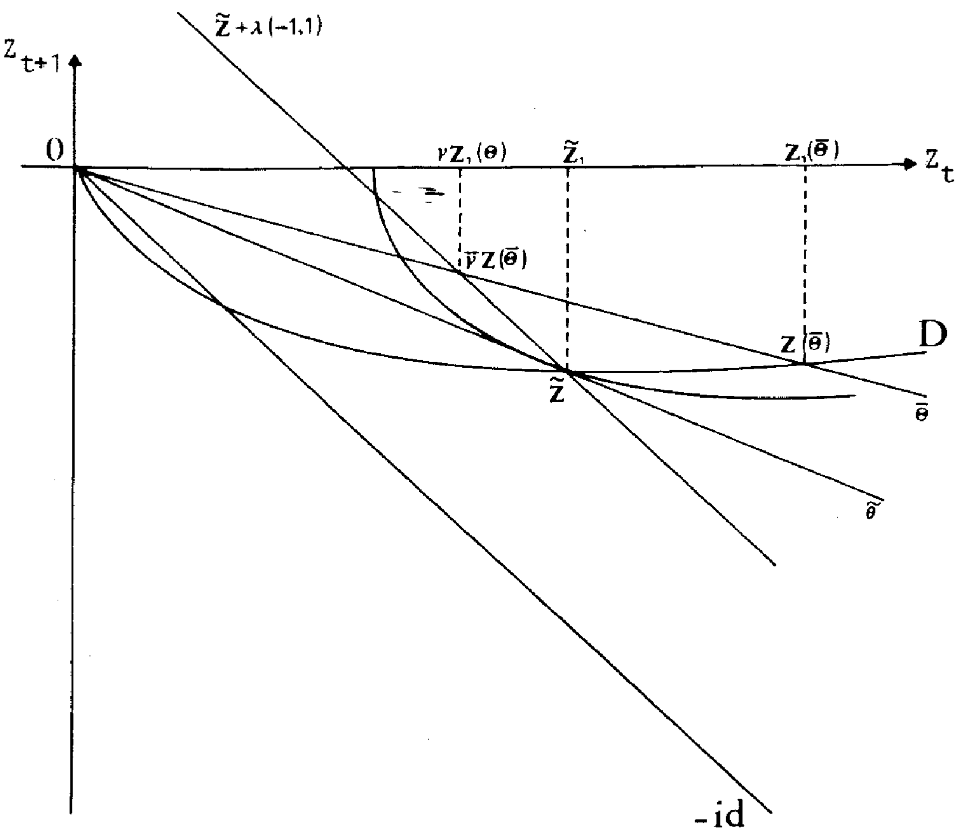

Figure 7

$\tilde{\theta}$ has been added. At $\bar{\Theta} < \tilde{\theta}$, $Z(\bar{\Theta})$ is the unconstrained demand point, whereas $\bar{\gamma}Z(\bar{\Theta})$ is the quantity constrained demand given the rationing level $\bar{\gamma}Z_1(\bar{\Theta})$. Clearly, $\bar{\gamma}Z_1(\bar{\Theta})$ yields higher utility than $\tilde{Z}$ and it is feasible for ECONOMY 2 since $\bar{\gamma}Z(\Theta) = (\tilde{Z}_1 - \bar{\lambda}, \tilde{Z}_2 + \bar{\lambda})$.

An interpretation of THEOREM 4 combined with PROPOSITION 3 is straightforward. Taken together with the findings of Section 3, inflation in international trade favors the debtor country on two counts. First, a non-trivial inflationary equilibrium yields higher utility to the debtor country, ECONOMY 2 in our case, than at the autarchy equilibrium. Second, the debtor country can increase the utility of its agents even more by running a higher inflation rate than the creditor country. To do so the debtor country may have to use quantity constraints which may be either binding demand constraints for the young or binding supply constraints for the old agents. The higher inflation rate implies that the exchange rate $\omega_t = (k/K)(\theta/\Theta)^{1-t}$ tends to zero as $t \to \infty$, i.e. the debtor country devalues its currency at a constant rate. Most importantly, however, such a policy can be carried out without requiring changes in the creditor country. Hence, as long as the creditor country does not counteract, the debtor country has a strong incentive to devaluate.

6. ECONOMIC POLICY, EXCHANGE RATES, AND THE BALANCE OF PAYMENTS

The formal structure of the model and its properties, as presented in the preceding sections, is all that is necessary to understand the welfare implications of a dynamic trade model in the overlapping generations framework. However, it is useful and informative to discuss the institutional implications and economic interpretations and relate them to some of the approaches and findings in monetary trade theory. Although most of the characterizations which follow also pertain to non-quasi-stationary equilibria, the presentation will be carried out exclusively using quasi-stationary equilibria.

Let $\big((x,X),(\theta,\Theta)\big)$ denote a quasi-stationary international equilibrium, where $1/\theta$ and $1/\Theta$ are the inflation rates in the two economies. Then, prices (p_t) and (P_t), $t \geqq 1$, in the two economies can be written as

$$\text{(i)} \qquad \begin{aligned} p_t &= k\theta^{1-t} \\ P_t &= K\Theta^{1-t}, \end{aligned}$$

where k and K are positive constants which are proportional to the initial quantity of money $m_1 > 0$ of ECONOMY 1 and to the initial level of debt $M_1 < 0$ of ECONOMY 2. Define the exchange ω_t, $t \geqq 1$ as $\omega_t = p_t/P_t$ and the balance of trade (i.e. current account) for the two economies as

$$\text{(ii)} \qquad \begin{aligned} b_t &= p_t(x_1 - e_1 + x_2 - e_2) \\ B_t &= P_t(X_1 - E_1 + X_2 - E_2), \end{aligned}$$

i.e. $b_t(B_t)$ is the value of aggregate excess demand of ECONOMY 1 (ECONOMY 2), i.e. the value of excess demand of the two generations living in each country at date t. Budget balance in each country implies

$$\text{(iii)} \qquad \begin{aligned} \theta(x_1 - e_1) + x_2 - e_2 &= 0 \\ \Theta(X_1 - E_1) + X_2 - E_2 &= 0, \end{aligned}$$

and feasibility requires

(iv) $\quad x_1 - e_1 + x_2 - e_2 + X_1 - E_1 + X_2 - E_2 = 0.$

(ii) and (iv) together imply the usual identity

(v) $\quad b_t + \omega_t B_t = 0.$

Let $m_t(M_t)$ denote the amount of money (debt) present in period t, i.e. the amount held over from period t-1 by generation t-1 to be spent (redeemed) in period t when old. Then (ii) and (iii) imply the second pair of identities

(vi)
$$b_t + m_{t+1} - m_t = 0$$
$$B_t + M_{t+1} - M_t = 0.$$

Let $\left((x,X)(\theta,\Theta)\right)$ be a stationary international equilibrium, i.e. $\theta = \Theta = 1$. Then $B_t = b_t = m_t - m_{t+1} = M_t - M_{t+1} = 0$ and $p_t = k$ and $P_t = K$ for all $t \geqq 1$. Hence, all lending and borrowing is done within each country at constant levels of money and debt. Moreover, no international trade occurs. This last property, however, is specific to the one commodity case and will not be true in general if there is more than one commodity in each period. As a consequence of these statements one has that non-zero trade balances occur if and only if the quantity of money and debt in the two countries is non-stationary which is possible if and only if θ and Θ are different from 1. These results follow immediately from (ii) and (iii).

In general, one has

(vii)
$$b_t = k(x_1 - e_1 + x_2 - e_2)\theta^{1-t}$$
$$B_t = K(X_1 - E_1 + X_2 - E_2)\Theta^{1-t}$$

and

(viii)
$$m_t = k(x_2 - e_2)\theta^{1-t}$$
$$M_t = K(X_2 - E_2)\Theta^{1-t}.$$

Moreover, $\omega_t = (k/K)\cdot(\theta/\Theta)^{1-t}$. Let $\big((x,X)(\theta,\Theta)\big)$ be an inflationary competitive equilibrium, i.e. $\theta = \Theta < 1$. Assume that $x_1 - e_1 < 0 < X_1 - E_1$, i.e. young agents of ECONOMY 1 are net savers and young agents of ECONOMY 2 are net borrowers. Then, all lending and borrowing is carried out between generations of the same age and across countries, since

$$x_1 - e_1 + X_1 - E_1 = 0.$$

However, at the constant exchange rate $\omega_t = k/K$, the trade surplus of ECONOMY 1 and the trade deficit of ECONOMY 2 tend to infinity at the inflation rate $1/\theta$. (viii) implies that the money stock m_t and the total debt M_t tend to infinity as well at the inflation rate. For this to be possible there must exist a monetary institution or policy in each country adjusting m_t and M_t appropriately.

Finally, consider an international equilibrium with price disparities, i.e. $\theta \neq \Theta$, and assume as before that $x_1 - e_1 < 0 < X_1 - E_1$. Price disparities imply that intercountry <u>and</u> intergenerational lending and borrowing will exist. Moreover, the exchange rate ω_t will tend to zero or to infinity depending on whether $\theta > \Theta$ or $\theta < \Theta$, i.e. $\theta > \Theta$ implies a devaluation of ECONOMY 2's currency at the rate Θ/θ. Hence, in such a setting flexibility of exchange rates as well as monetary policies are needed to sustain equilibria with price disparities. Total money and debt will tend to infinity at different rates if there is inflation in both countries. It is important to note that quasi-stationary equilibria provide only a limited role for the exchange rate as an equilibrating mechanism for the balance of trade. This is easily seen from (vii). Since intertemporal prices, i.e. θ or Θ, determine the net savings position of any young agent, the sign and the magnitude of the balance of trade is strictly a function of intertemporal price ratios in each country and not of the intercountry price ratio ω_t.

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